

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT2/4TE01EMT3

Branch: B.Tech (All)

Semester: 1

Date: 09/03/2021

Time: 03:00 To 06:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: [14]

a) n^{th} derivative of $y = \frac{1}{x+a}$ is _____ (01)

a) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ c) $\frac{(-1)^n n!}{(x+a)^n}$

b) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ d) None of these

b) Let y_n denotes the n^{th} derivative of y , where $y = e^{-x}$ then $y_n =$ _____ (01)

a) $-e^{-x}$ c) $(-1)^{n+1} e^{-x}$

b) $(-1)^n e^{-x}$ d) None of these

c) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ represent expansion of _____ (01)

a) $\sin xc)$ $\sinh x$

b) $\cos xd)$ $\cosh x$

d) $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \frac{y^6}{6!} \dots$ is the expansion of _____ (01)

a) $\cos x$ c) $\sin x$

b) $\cos hx$ d) $\sin hx$

e) If $y = \cos^{-1} x$ then $x =$ _____ (01)



- a) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ c) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} - \dots$
 b) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$ d) None of these

f) Find $\lim_{x \rightarrow \infty} x^4 e^{-ax}$ (01)

- a) 4 c) -4
 b) 0 d) 1

g) Find the incorrect relation from following. (01)

- a) $\sin ix = i \sin hx$ c) $\tanh ix = i \tan x$
 b) $\cos ix = -\cos hx$ d) $\sinh ix = i \sin x$

h) If $u = y^x$ then $\frac{\partial u}{\partial x}$ is ____ (01)

- a) xy^{x-1} c) $y^x \log x$
 b) 0 d) None of these

i) If $x = r \cos \theta$ and $y = r \sin \theta$ then $\frac{\partial r}{\partial x} =$ ____ (01)

- a) $\sec \theta$ c) $\operatorname{cosec} \theta$
 b) $\sin \theta$ d) $\cos \theta$

j) If $u = ax^2 + 2hxy + by^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ ____ (01)

- a) $2u$ c) 0
 b) u d) None of these

k) The conjugate of $z = 1 - 3i$ is ____ (01)

- a) $1 - 3i$ c) $3i$
 b) $1 + 3i$ d) 1

l) Find $\det A$ where $A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$. (01)

m) Write an example of Upper Triangular Matrix. (01)

n) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -8 & 0 & -1 \end{bmatrix}$ then find A^T . (01)

Attempt any four questions from Q-2 to Q-8.

Q-2 Attempt all questions [14]

a) Let $y = \cos x \cos 2x \cos 3x$ then find y_n . (05)

b) If $y = (1 - x^2)^{-\frac{1}{2}} \sin^{-1} x$, then show that
 $(1 - x^2)y_{n+1} - (2n + 1)xy_n - n^2y_{n-1} = 0$ where y_n denotes n^{th}
 derivative of y . (05)



c) Find n^{th} derivative of $y = \log(ax + b)$. (04)

Q-3 Attempt all questions [14]

a) Expand $f(x) = \sec x$ in powers of x up to x^4 by Maclaurin's series. (06)

b) Using Taylor's series expansion, expand $\log x$ in powers of $(x - 2)$. (05)

c) If $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ then show that $x = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \dots$ (03)

Q-4 Attempt all questions [14]

a) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$. (05)

b) Evaluate $\lim_{x \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + x^2}{x^3}$ (05)

c) Find y where $y = \lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$ (04)

Q-5 Attempt all questions [14]

a) For $u = \tan^{-1}\left(\frac{y}{x}\right)$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ (05)

b) f is homogeneous function of degree 2 defined by (05)

$$f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2} \text{ then show that } x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + 2f(x, y) = 0$$

c) Find $\frac{\partial^2 u}{\partial t^2}$ and $\frac{\partial^2 u}{\partial x^2}$, where $u = e^{x-at} \cos(x - at)$. Is there any relation between $\frac{\partial^2 u}{\partial t^2}$ and $\frac{\partial^2 u}{\partial x^2}$? (04)

Q-6 Attempt all questions [14]

a) Using De-Moivre's theorem prove the following: (06)

$$1) \cos 5\theta = 5 \cos \theta - 20 \cos^3 \theta + 16 \cos^5 \theta$$

$$2) \sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$$

b) If $\sin(\alpha + i\beta) = x + iy$ then prove that $x^2 \operatorname{cosec}^2 \alpha - y^2 \sec^2 \alpha = 1$. (04)

c) Find $\tanh x$ if $5 \sinh x - \cosh x = 5$. (04)

Q-7 Attempt all questions [14]

a) Solve the system of linear equations: (06)

$$x + 2y = 3, y - z = 2, x + y + z = 1$$

b) Find the rank of $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$. (05)

c) Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (03)

Q-8 Attempt all questions [14]

a) Using De Moivre's theorem expand $\sin^8 \theta$ in a series of cosines of multiple of θ . (06)

b) For real value of z , show that $\sinh^{-1} z = \log(z + \sqrt{z^2 + 1})$ (04)

c) Simplify: $\frac{(\cos 3\theta + i \sin 3\theta)^{-2} (\cos 2\theta - i \sin 2\theta)^{\frac{3}{2}}}{(\cos 5\theta - i \sin 5\theta)^3 (\cos 2\theta + i \sin 2\theta)^7}$ (04)

